

# Towards an Observational Appraisal of String Cosmology

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**Abstract.** We review the current observational status of string cosmology when confronted with experimental datasets. We begin by defining common observational parameters and discuss how they are determined for a given model. Then we review the observable footprints of several string theoretic models, discussing the significance of various potential signals. Throughout we comment on present and future prospects of finding evidence for string theory in cosmology, and on significant issues for the future.

## 1. Introduction

Thanks to important advances in experimental astrophysics, the past two decades has seen modern cosmology become a high precision, data rich science (see for example [1, 2, 3, 4, 5, 6, 7, 8, 9]). Observations suggest that our universe is consistent with a flat, so called ‘ $\Lambda$ CDM universe’, and have confirmed that our favoured theory for the origin of structure, cosmological inflation [10] is well supported by the data.

Given the extremely high energy scales present in the early universe, and huge distances probed by large scale cosmological evolution, cosmological observables may be sensitive to Planck scale physics. String theory is a leading contender for a theory of the ultraviolet (UV) completion of quantum field theory and gravity, and hence one within which Planck scale effects can be addressed. It is natural, therefore, to ask what the consequences of cosmology are for string theory, and visa versa. Thus the notion of ‘string cosmology’ [43] was born.

Ideally string cosmology is the *direct* application of string theory to understanding the evolution of the universe. Then, through comparison with data, we might hope to experimentally test the theory. In practice, however, this is too ambitious and generally models are constructed using ideas and intuition from string theory, which are then confronted with observation. Although such a program has been remarkably successful, the inherent stringiness of the models is seldom explicitly present, and it is difficult to argue that constraining models is a direct probe of string theory itself.

On the other hand, particular theories can be sensitive to UV physics, even if not probing it directly, and certain self-interaction potentials for the inflaton can arise in string motivated models which are rather unlikely to arise from pure field theory constructions. Moreover, the self consistency of string inflation models can certainly be probed by observation. Furthermore, while a less developed area of research, string cosmology extends beyond the inflationary epoch. It may, for example, offer convincing alternatives to inflation, which flow more directly from the UV complete nature of the theory. It is also possible to generate cosmic *superstrings* which could be directly detected by observation. String theory may even have a role to play in understanding why our universe is accelerating today.

With all this in mind, the purpose of the present review is to ask how far we have come in our quest to probe string theory using cosmology, and to address questions such as: will we ever see observational signatures of string theory in cosmology? And what are the most promising signals to look for?

We structure the review as follows. In Section 2, we review how inflationary models are confronted with observation, and the strength of present and future constraints, as well as discussing other observations relevant to probing string theory. In Section 3, we discuss how string theory models are being tested by observation, discussing what would constitute evidence of string theory in light of the issues of naturalness and robustness of models, and review a number of inflationary models and others together with their observational predictions. Finally we conclude in Section 4 by highlighting what the key issues are for the future.

## 2. Observations and discriminators of early universe models

There exists an extremely well developed framework for determining how well a given inflationary model (or alternative) fits experimental data, which we will now review. We then discuss current and future observational constraints, as well as other observations of interest for string cosmology.

### 2.1. Discriminators of the very early universe

Inflation is the accelerated expansion of spacetime; during which quantum fluctuations of the metric and matter are ‘stretched’ to large scales, and subsequently become the origin of cosmic structure (for useful reviews see for example [17, 18, 19, 20]). The fundamental scalar quantity is the primordial curvature perturbation  $\zeta$  and tensor fluctuations are parametrised by their amplitude,  $T$  (see for example [23, 21]). Isocurvature perturbations may also be produced and persist after inflation, but are not inevitable and are disfavoured by current data. In the absence of isocurvature perturbations, a given wavenumber,  $k$ , of  $\zeta$  is conserved once stretched to super-horizon scales,  $k < aH$ , where  $a$  is the scalar factor and  $H = \dot{a}/a$  [23, 22, 24, 26, 25]. The stochastic properties of these perturbations are probed by observation. The full power spectrum, parametrisation

the two-point function, can be a powerful tool, but is generally parametrised about some pivot scale,  $k_*$  in terms of a number of key parameters. The first are the square of the amplitude of the scalar and tensor modes, denoted  $\mathcal{P}_\zeta^*$  and  $\mathcal{P}_T^*$  respectively, which lead to the ratio  $r = \mathcal{P}_T^*/\mathcal{P}_\zeta^*$ . This parameter is important because a detection would directly probe the energy scale of inflation. The next key parameters are the spectral tilts;  $n_s(k) - 1 = d \ln \mathcal{P}_\zeta / d \ln k|_*$  for scalar perturbations, and  $n_T(k) = d \ln \mathcal{P}_T / d \ln k|_*$  for tensors. One can then continue to define a running, the derivative of the tilt with respect to  $\ln k$ , and higher derivative parameters if required.

Further information is available from studying statistics beyond the two-point function. The three-point function, which vanishes for Gaussian perturbations, is parametrised by the bispectrum,  $B_\zeta(k, k', k'')$ , generally normalised by the square of the power spectrum to give the reduced bispectrum or  $f_{\text{NL}}(k, k', k'')$  parameter. One can then continue to the trispectrum (four-point function), running of  $f_{\text{NL}}$  and so on. Currently meaningful constraints exist only for the subset of parameters  $\{r, n_s, f_{\text{NL}}\}$  and the running of  $n_s$ .

Single field inflationary models are the most widely studied, because of their simplicity, and are characterised by the generalised action

$$S = \int d^4x \sqrt{-g} \left[ \frac{m_{pl}^2}{2} R + P(\phi, X) \right], \quad (2.1)$$

where  $R$  is the Ricci scalar, minimal coupling has been assumed and  $X = -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ . For inflation driven by this action it has been found that [30]

$$\mathcal{P}_\zeta = \frac{1}{8\pi^2 m_{pl}^2} \frac{H^2}{c_s \epsilon}, \quad \mathcal{P}_T = \frac{2}{\pi^2} \frac{H^2}{m_{pl}^2} \quad (2.2)$$

where  $\epsilon = -\dot{H}/H^2$ ,  $c_s$  is sound speed of scalar fluctuations,  $c_s^2 = P_{,X}/(P_{,X} + 2XP_{,XX})$ , and all expressions are evaluated at the scale  $k^* = aH/c_s$ . We note that accelerated expansion requires  $\epsilon < 1$ , and  $c_s = 1$  corresponds to an canonical scalar field with modes travelling at the speed of light, for which  $\epsilon \approx \frac{m_{pl}^2}{2}(V'/V)^2$ . Moreover one finds

$$r = 16c_s\epsilon, \quad (1 - n_s) = 2\epsilon + \frac{\dot{\epsilon}}{\epsilon H} + \frac{\dot{c}_s}{c_s H}, \quad n_t = -2\epsilon \quad (2.3)$$

and for equilateral triangles  $k \sim k' \sim k''$  [27, 28, 29]

$$f_{\text{NL}}^{\text{eq}} = \frac{5}{81} \left( \frac{1}{c_s} - 1 - 2\Lambda \right) - \frac{35}{108} \left( \frac{1}{c_s} - 1 \right), \quad (2.4)$$

where,  $\Lambda = (X^2 P_{,XX} + \frac{2}{3} X^3 P_{,XXX}) / (X P_{,X} + 2X^2 P_{,XX})$ . All shapes of  $f_{\text{NL}}$  are negligible for canonical single field models with  $P = X$ .

To predict the observational parameters from a given model of inflation, we must find the values that parameters ( $\epsilon$ ,  $c_s$  etc.) took when  $k^* = aH/c_s$ . This is dependent on the number of e-folds  $\mathcal{N}^* = \ln(a_{\text{end}}/a^*)$ , which in turn depends on post inflationary physics. A reasonable range is  $\mathcal{N}^* \approx 54 \pm 7$  [33], but values *well* outside this range are possible. Since the observational footprint of any given model will be sensitive to  $\mathcal{N}^*$ ,

to properly compare a given model with observation one must generally determine its *post-inflationary* behaviour. Unfortunately this is not known in most models.

Space will not permit a careful review of how observational predictions are made for every string cosmology model we will discuss, so here, following [31, 32] (where the reader can turn for a fuller discussion of observational constraints), we include two illustrative examples of canonical models (which cover a large number in the literature):

$$(a) \ V = V_0 \left[ 1 - \left( \frac{\phi}{\lambda} \right)^p \right], \quad (b) \ V = V_0 \left( \frac{\phi}{\lambda} \right)^p, \quad (2.5)$$

where  $V_0$ ,  $\lambda$  and  $p$  are constants. Assuming the potential maintains this form over the entire inflationary evolution, and using the approximate expression  $d\phi/d\mathcal{N} \approx -\sqrt{2\epsilon}m_{pl}^2$ , which follows when  $\epsilon \ll 1$ , one can find the field value and thus all relevant quantities  $\mathcal{N}^*$  e-folds before the end of inflation (defined as  $\epsilon = 1$ ). Potential (a) represents a small field model for which the range of field values traversed during inflation is  $|\Delta\phi| < m_{pl}$ , and (b) represents a large field model where  $|\Delta\phi| > m_{pl}$ , raising the usual issue of corrections to the potential. Following the procedure outlined, for (a) one finds that  $r$  is negligible (typical of small scale models since one can in general express  $r = 8(d\phi/d\mathcal{N})^2/m_{pl}^2$  [42]), and  $n_s = 1 - 2 \left( \frac{p-1}{p-2} \right) \frac{1}{\mathcal{N}^*}$  (the case of  $p = -\infty$  corresponds to the potential  $V = V_0(1 - e^{-a\phi/m_{pl}})$ , and  $p = 0$  to  $V = V_0(1 + A \ln(\phi/B))$ ). For (b) one finds  $1 - n_s = \frac{2+p}{2\mathcal{N}^*}$  and  $r = \frac{8[\mathcal{N}^*(1-n_s)-1]}{\mathcal{N}^*}$  (where we have considered  $p > 0$ ). The relation between  $n_s$  and  $r$  is important because the corresponding parameter space is well constrained. Note, however, that such simple expressions follow from the simplicity of the potential. Were there additional (potentially unknown) terms, such simple relations would not exist. There are, therefore, two key lessons of this discussion. Firstly observables depend on  $\mathcal{N}^*$ , which we don't know apriori and is not itself an observable. And secondly, simple relations between parameters are possible but will be spoilt if the form of the potential is altered by further terms arising from quantum corrections, and which may introduce new parameters.

Further complications arise if more than one field is light at horizon crossing, since isocurvature modes will be produced. No observational evidence that such modes existed has been found, but were it to be it would rule out single field inflation. When isocurvature modes are present, the curvature perturbation and its statistics may evolve on super-horizon scales during inflation if the field space path curves [34]. Even if isocurvature modes decay before or during reheating, a curved path during inflation will alter the relation between observational parameters and the value of  $\epsilon^*$  etc. at horizon crossing (even if the path only curves after modes around the pivot scale exit the horizon). The best developed method to account for this is the  $\delta\mathcal{N}$  formalism [35, 36, 37]. Space restricts us from providing the full details, but for canonical inflation the amplitude of the power spectrum is given by  $\mathcal{P}_\zeta = \mathcal{N}_{,i}\mathcal{N}_{,i}H^2/(4\pi^2)$ , the tilt by  $1 - n_s = 2\epsilon^* - 2\dot{\phi}_i^* \mathcal{N}_{,ij}\mathcal{N}_{,j}/(H^*\mathcal{N}_{,k}\mathcal{N}_{,k})$ , and the most important contribution to the reduced bispectrum in the squeezed limit ( $k \sim k' \ll k''$ ) is  $f_{NL}^{loc} = 5/6 \mathcal{N}_{,i}\mathcal{N}_{,ij}\mathcal{N}_{,j}/(\mathcal{N}_{,k}\mathcal{N}_{,k})^2$ , which can be large for certain models. Here  $\mathcal{N}$  is

the number of e-folds from horizon crossing to a constant energy density hypersurface once the evolution has become adiabatic, roman numerals label the  $\mathcal{M}$  light fields, and the subscripts denote derivatives with respect to changes in the field values at horizon crossing. In multi-field models, therefore, simple relations for quantities such as  $n_s$  are only available for the simplest trajectories, and moreover, a  $\mathcal{M} - 1$  dimensional surface now leads to any given  $\mathcal{N}^*$ .

## 2.2. Observational constraints and other signatures

Observations of the CMB constitutes the most important tool at our disposal to constrain the defined parameters. Normalisation of the CMB anisotropy requires  $\mathcal{P}_\zeta = 2.42 \times 10^{-9}$  [5]. Precise constraints on the observational parameters depend on how many parameters are included in the statistical analysis, and what other data sets are included. For example, at the 68% confidence level, if the running of the scalar spectral index and  $r$  are assumed to be zero, the WMAP-7 [5] data alone implies  $n_s \approx 0.967 \pm 0.014$ . If  $r$  is also included the data gives  $n_s \approx 0.982 \pm 0.02$  and  $r < 0.36$  (at the 95% confidence level), while allowing for running of the scalar spectral index leads to  $n_s \approx 1.027 \pm 0.05$  and  $dn_s/d\ln k \approx -0.034 \pm 0.026$  ( $r$  taken to be zero). Moreover, it is important to recognise that the analysis assumes the absence of other contributions to density fluctuations, such as cosmic (super)strings. When these were included, a recent study found a blue spectrum ( $n_s > 1$ ) could be accommodated [122] ( $n_s = 1.00 \pm 0.03$  with a maximum 11% contribution to power from cosmic strings). An important outcome, therefore, is to recognise that while statements such as the WMAP data favours a red ( $n_s < 1$ ) spectrum are common, this is highly analysis dependent.

WMAP also constrains non-Gaussianity, with  $-10 < f_{\text{NL}}^{\text{loc}} < 74$  and  $-214 < f_{\text{NL}}^{\text{eq}} < 266$  at the 95% confidence level.

The Planck satellite [11] currently taking data will hopefully improve these constraints considerably, and in particular, in the absence of a detection one expects limits of roughly  $|f_{\text{NL}}| < 5$ , and  $r < 0.05$  and error bars on  $n_s$  at least an order of magnitude better than WMAP. The proposed CMBPOL mission [38] (designed specifically to look at the polarisation of the CMB) could probe down to  $r < 0.01$ . There are other exciting future possibilities, such as observation of 21cm radiation [12], which probes the structure of the universe during the cosmic dark ages before re-ionisation, and could give limits of  $|f_{\text{NL}}| < 1$ . Other important observations which constrain primordial perturbations over different scales are the various surveys of large scale structure, which are often combined with WMAP data, and in particular can give constraints on the running of parameters (see for example [13]).

CMB polarization is also an important discriminator in its own right. Scalar perturbations generate only E-modes, whilst tensor perturbations generate both E and B-modes. Vector perturbations also generate B-modes (the E-mode being negligible with respect to the B-mode), and while highly suppressed during inflation are sourced by cosmic strings. Thus the detection of B-modes would automatically lead to exciting

new knowledge about the universe. One caveat is that we must assume there is no axionic coupling to the photon through terms of the form  $\sigma a F \wedge F$ , since this can rotate the E-mode into a B-mode with mixing angle given by  $\Delta\theta \sim \sigma \Delta a$ . Current WMAP bounds on this angle at the 68% confidence level are

$$\Delta\theta = -1.1\text{deg} \pm 1.4\text{deg}(\text{stat}) \pm 1.5\text{deg}(\text{syst}) \quad (2.6)$$

which is consistent with it vanishing, but further observations are clearly required <sup>‡</sup>.

Observational cosmology is an incredibly rich field, and important data for string cosmology may lurk in numerous other observations. As we will discuss, gravitational lensing - both strongly lensed images and weak lensing - could contain information about cosmic strings, as could micro lensing surveys [16]. Moreover, data is available on the peculiar velocities of clusters of galaxies, which probe the laws of gravity on the largest scales [41]. Not to mention the improving data from supernovae which played the crucial role in determining the need for a dark energy component (acting like a cosmological constant) to accelerate the universe today [39]. The potential evolution of dark energy will be a key focus of future observations. Finally, we note there is potential for the direct observation of gravitational waves by ground or space based interferometers (LIGO and LISA) [14, 15]. Though we will likely have to wait many years before an experiment has the sensitivity to be relevant to inflationary gravitational waves (BBO) [40], constraints relevant to cosmic strings already exist [113].

### 3. Testing string theory using cosmology

The overall goal of the string cosmology program is to use cosmology as a testing ground for string theory. The twin aims are to understand whether and how string theory constructions can explain observed properties of the universe, and, more excitingly, to determine whether there might be a *signal* of string theory in observations. The holy grail would be an observational confirmation through the direct detection of something genuinely ‘stringy’. It is possible, perhaps even likely, though that there will be no smoking gun, rather, evidence for string theory might come in a less dramatic form; for example by providing a natural, convincing explanation for something already observed but not properly understood. An example would be a truly compelling model of inflation (or an alternative). It may transpire, however, that all we can achieve is consistency of models with observables, and nothing more. Yet even if this were so, a well motivated string inflationary model which passed successive observational tests of this type, at the expense of other models failing, could come to be seen as evidence of string theory itself. Likely such evidence would need to be augmented by other observations. For example if such a model additionally predicted cosmic superstrings and some evidence of cosmic strings was found, it would become doubly appealing.

In light of these points, as we discuss aspects of string cosmology – and models of the early universe in particular – we will keep three issues in mind. The first is

<sup>‡</sup> Such a coupling can arise in string models through the inclusion of Wess-Zumino (WZ) terms.



naturalness of the model. This is a hard concept to make precise but, for example, a model for which a vast range of parameters or initial conditions is allowed, but for which only a vanishingly small range leads to a consistent cosmology, could be viewed as unnatural. The second is testability. Is the model sufficiently well developed that all its parameters are derivable and constrained by observation? Or does it require elements that are plausible, but not yet consistently implemented? Ideally to test models we should have more observable parameters than model parameters, if not we can only probe combinations of model parameters. Finally a model could be predictive, in the limited sense that it leads to a ‘stringy’ prediction, which could not come from, or would be hard to produce, in the absence of stringy physics.

### 3.1. String Inflation

First we consider string inflationary models. We aim to review a representative selection, discuss how they confront observation and attempt to address how well they measure up against the issues of naturalness, testability and predictivity. We will be particularly interested in those that include concrete calculations of world-sheet or loop corrections, since these are both a sign of the maturity of a model, and will likely lead to reliable signals and consistency relations. We will also focus on observable features which seem to appear more naturally in a string theory setting.

Before embarking on this path, we first note that while many parameters appearing in a model, such as background fluxes, are inherently stringy, we cannot measure them directly cosmologically because we restrict ourselves to an effective inflaton action. Such parameters are absorbed by field redefinitions, giving rise to additional degeneracies, and it is difficult to argue that by probing a given model we are truly probing the original stringy parameters. Moreover, we adopt a critical position that although supergravities are the low energy limits of string theory, an observable signature within supergravity is not (in itself) proof of a string theory signal.

*Modular inflation.* Most work in this area has focused on flux compactification of type IIB supergravity. Such compactifications preserve at least one, but in general many, massless moduli due to the no-scale structure of the classical theory. These fields can acquire a mass through non-perturbative contributions to the superpotential, such as those arising from gaugino condensation on wrapped  $D$ -branes. However such terms can only be calculated explicitly in string theory in a handful of cases, since they typically depend on moduli that have been integrated out of the theory. The first attempt to fix all moduli was the KKL $T$  construction [44] which considered one complex modulus. The resulting potential requires an additional uplifting term provided by an anti- $D3$  ( $\bar{D}3$ ) brane at the tip of the warped throat, to obtain a  $dS$  vacuum, and doesn’t lead to a consistent inflationary scenario, but the basic procedure underlies many other models, and including more than one moduli can lead to viable inflationary models.

One such scenario, tree level in loops but including world-sheet corrections, is Racetrack inflation on the  $\mathbb{CP}^4[1, 1, 1, 6, 9]$  Calabi-Yau (CY) three-fold [47, 48]. The

complicated ‘Racetrack’ potential arises from competition between competing terms in the non-perturbative superpotential, with two scalar fields driving inflation [46]. Inflation is possible with particular initial conditions, but not generic, on this potential and satisfying the WMAP normalisation of the power spectrum proves challenging. The authors restricted themselves to variations of the constant term in the superpotential ( $W_0$ ), and found that maximising the manifold volume led to  $n_s \sim 0.95$  for a straight trajectory evolving from the saddle point and emulating a small field model, with negligible  $f_{\text{NL}}$  and  $r$ . While a useful proof of principal, meaningfully probing the parameter space using observations for this model would be extremely difficult.

Another model of interest is based on the large volume scenario (LVS) [53, 45] with one or more of the geometric moduli identified as the inflaton  $\S$ . The world-sheet corrections ensure that the volume is stabilised at exponentially large values after inflation. There are several different models in this class including Kähler inflation, Roulette inflation [50] and Fibre inflation [49] which all have exponential potentials. Both Kähler and Roulette inflation lead to small tensor models  $r \sim \mathcal{O}(10^{-10})$ , with the simplest Kähler model having the form  $V = V_0(1 - e^{-a\phi})$ , and hence leads to  $n_s = 1 - 2/\mathcal{N}^*$  (this corresponds to the  $p = -\infty$  small field example of Section 2.1). In Roulette models the inflaton is associated with a classical trajectory through field space (perpendicular to the isocurvature trajectory) [50]. However inflationary trajectories typically have  $\epsilon^* \sim 0$ , which suggests conflict with the WMAP data. In the more general multi-field scenario, results indicate a larger red-tilted power spectrum with  $|f_{\text{NL}}| < 0.1$ . Fibre inflation consists of a  $K3$ -manifold fibred over a  $\mathbb{CP}^1$ , allowing for the explicit inclusion of one-loop corrections. These corrections are quantified by  $R = 16AC/B^2$ , where  $A, C$  are terms explicitly arising from loop effects. Sufficient e-folds are obtained for small  $R$ , with  $\mathcal{N} = 60$  occurring for  $R = 3 \times 10^{-5}$ . Therefore in the limit that  $R \rightarrow 0$  one finds the model independent footprint $\parallel$   $r \simeq 6(n_s - 1)^2$  which is within current WMAP bounds. In the opposing regime we find  $r \simeq (32/3)R^{2/3}$ ,  $n_s \simeq 1 - 4R^{2/3}$  which implies  $r \leq 0.01$ ,  $n_s \leq 0.996$  at  $\mathcal{N} = 60$ . A two-field model was also constructed in this class with similar results [49].

These large volume models are well motivated, considered natural, and are testable at least for single field constructions, though they do not yet predict any genuinely stringy signatures. One should note that loop corrections are not universal, although their general form is known [51], and must be computed for each CY. The prototypical case is  $\mathbb{CP}^4[1, 1, 1, 6, 9]$ , where the leading order perturbative and non-perturbative world-sheet corrections, and the one and two-loop terms are known [52]. The non-perturbative corrections ensure a  $dS$  vacuum without the need for  $\bar{D}3$  branes, however the loop corrections were argued to be *sub-dominant* with respect to the world-sheet corrections. The inflaton here [52] is a linear combination of NS-NS axions, and inflation occurs at a saddle-point where  $\mathcal{N}$  depends explicitly on the degree of the genus-zero holomorphic curve. Subsequent work computed cosmological observables, which are

$\S$  This is a Kähler modulus in IIB and a complex structure modulus in IIA.

$\parallel$  Meaning that the observables only depend on the slow roll parameters.



sensitive to the volume ( $\mathcal{V}$ ) and the  $D3$ -instanton number ( $n^s$ ). Favourable scenarios require  $\mathcal{V} \gg 1$ ,  $n^s \sim 10$ , which yields  $|f_{nl}| \sim 10^{-2}$ ,  $r \sim 10^{-4}$  and  $|n_s - 1| \sim 10^{-3}$  [52]. Although one can compute various (soft) susy-breaking masses and Yukawa couplings, which are themselves expected to be experimentally constrained, direct cosmological observation will require the inclusion of loop corrections to distinguish these models from field theory.

### 3.2. Brane inflation models

$D$ -branes play a key role in modern string theory, so it is natural to consider their cosmological consequences. These branes are described by non-linear Dirac-Born-Infeld (DBI) theory. The most popular class of cosmological models exist in IIB, where a mobile  $D$  brane travels relativistically down a warped throat towards attractive  $\bar{D}3$ -brane charge [62]. In the simplest case  $P(\phi, X)$  is of the form

$$P = -T(\phi)\sqrt{1 - 2XT(\phi)^{-1}} + T(\phi) - V(\phi) \quad (3.7)$$

with  $T$  the warped brane tension and  $V(\phi)$  the scalar potential. The non-linear nature of DBI-inflation ensures that  $c_s$  can become small, leading to large equilateral  $f_{NL}$  (Eq.2.4). These models are potentially testable [64], predictive and moreover the speed limit imposed by the warped geometry was originally thought to make inflation extremely natural. Combined, these features have lead to significant interest in such models. The brane, however, can only travel a finite distance  $\Delta\phi$  due to the finite length of the warped throat [65, 63], which translates to an upper limit on  $r$ , as discussed in Section 2. When implemented with the above  $D3$ -brane action, assuming the simplest throat constructions, one finds [63, 66]

$$r_* < \frac{32}{\mathcal{N}\mathcal{N}_{eff}^2}(c_s P_{,X})_* \quad (3.8)$$

where  $30 < \mathcal{N}_{eff} < 60$  is expected. Moreover, when combined with Eq.(2.4) and other observational constraints, this implies  $f_{NL}$  would have to be outside the WMAP bounds [66]. A result which is independent of the scalar potential. While this is disappointing, it highlights that we are now genuinely able to confront string theoretic models in an increasingly powerful way. Before calculations of  $f_{NL}$  were performed, and the WMAP constraints available, this model would have appeared viable at the level of the power spectrum. Now despite its appeal, in its simplest form at least, it can be ruled out. Note that small field DBI-inflation may evade this bound while still leading to an  $f_{NL}$  signal within reach of Planck (see for example [67]). Moreover complex models can be constructed which relax the above bounds and include, angular modes, wrapped branes, multiple branes and multi-field theories [68] - though the issue of naturalness must be raised in this context. One such proposal, independent of the scalar potential using multiple/wrapped branes, links the tensor-scalar ratio directly to  $f_{NL}$  via

$$r_* < -\frac{5}{\mathcal{N}_{eff}^2} \frac{f_{nl}}{(n-1)\sqrt{N}} \quad (3.9)$$

where  $N$  is the  $D3$ -brane charge of the  $AdS_5 \times X_5$  geometry, and  $n$  is the number of branes. Such a model is clearly ruled out if  $f_{nl}$  is observed to be zero, or has positive sign. In the case of wrapped  $D(3 + 2k)$ -branes, it was found that the backreaction becomes more important for higher  $k$ . The wrapped  $D7$ -brane bound becomes

$$\frac{(1 - n_s)}{8} < r < \frac{216\pi}{3^4} \frac{K^4 P_s^2}{\text{Vol}(X_5)(\Delta N)^6} \left(1 + \frac{1}{3f_{nl}}\right) \quad (3.10)$$

which can be satisfied for  $K \geq 46$ , where  $K \in \mathbb{Z}$  is the NS-NS flux at the tip of the throat. There are many possible extensions, and it is likely to be a major area of inflationary model building for some time to come.

Another open string model embedded into IIB compactifications is that of  $D3/D7$ -brane inflation [69, 70], where the compact manifold is  $K3 \times T^2/Z_2$ . The inflaton is related to the distance between the two types of brane on the orbifold, and its potential is generated by the presence of non self-dual flux on the  $D7$ -brane. This generates a non-zero D-term potential with Fayet-Illioupoulos (FI) parameter  $\xi$ . The resulting mechanism is a stringy version of hybrid inflation, and inclusion of loop effects leads to the footprint [70]

$$n_s = 1 - \alpha \left(1 + \frac{1}{(1 - e^{-\alpha N})}\right), \quad \alpha = \frac{4m^2}{g_s^2 \xi^2} \quad (3.11)$$

An interesting consequence of many brane models is the creation of cosmic superstrings, which is perhaps the most predictive of all possibly observable stringy physics. This model is a particularly interesting example for which strings with a tension spectrum  $G\mu \sim \xi/4$  are produced, and which could be used to constrain or fix the FI parameter. Under the assumption that cosmic strings contribute at most  $\mathcal{O}(11\%)$  to the power spectrum, this implies  $r < 10^{-4} g_s^2$ , which is vanishingly small for perturbative strings. Increasing the scalar mass tends to suppress the cosmic string contribution, but shifts  $n_s$  further towards unity, and towards unfavoured WMAP values. This is another interesting example of how combinations of observables can probe or constrain a model.

One interesting recent development in brane inflation models, has been to consider corrections to the inflaton potential, for brane models in which the brane is moving non-relativistically, from compactification effects in the throat [72, 71]. Such a calculation has been done for a class of the simplest  $D3/\bar{D}3$  models discussed above. At leading order the inflaton potential is generated by the Coulombic interaction between these branes, however the corrections due to a single angular mode can be included resulting in the following potential

$$V(\phi) = V_0(\phi) + M_p^2 H_0^2 \left( \left( \frac{\phi}{M_p} \right)^2 - a_\Delta \left( \frac{\phi}{M_p} \right)^\Delta \right), \quad a_\Delta = c_\Delta \left( \frac{M_p}{\phi_{UV}} \right)^\Delta \quad (3.12)$$

where  $V_0(\phi)$  generally includes all terms that yield negligible corrections to  $\eta$ . Note that  $\Delta$  corresponds to the eigenvalues of the compact Laplacian, and the smallest eigenvalue ( $\Delta = 3/2$ ) is expected to dominate. However if symmetries forbid the existence of such a term, the next possible contribution comes from quadratic modes  $\Delta = 2$ . Such a model is relatively generic in that it relies on the computation of the Laplacian in the

non-compact throat, rather than on the full details of the compact space. The potential in the case of  $\Delta = 2$  has been well studied since it is of the form  $V(\phi) = V_0(\phi) + \beta H_0^2 \phi^2$ . Slow roll inflation requires  $\beta \ll 1$ , because the potential becomes steep as  $\beta \rightarrow 1$  ¶. The inflationary footprint is

$$n_s - 1 \sim \frac{2\beta}{3} \left( 1 - \frac{5}{(e^{2\beta\mathcal{N}} - 1)} \right) \quad (3.13)$$

with  $r(\beta \sim 0.1) \sim 10^{-4}$ , significantly larger than the KKLMMT model where  $r \sim 10^{-9}$  [72]. The full phenomenology is discussed in [73] where they used the WMAP data to bound the parameter  $\beta$ , however for fully UV complete scenarios we expect this to be fixed.

Finally we mention a  $D$ -term inflationary model in IIA, arising from the intersection of four brane stacks in a phenomenological configuration [61]. The inflaton connects two different brane stacks, and has a one-loop potential of the form

$$V(\phi) = g^2 \xi^2 \left( 1 + \frac{g^2}{4\pi^2} \ln \left( \frac{\lambda \phi^2}{\Lambda^2} \right) \right) \quad (3.14)$$

with scalar index  $n_s = 1 - 1/\mathcal{N}^*$  (this is the  $p = 0$  small field example of Section 2.1). The FI-term  $\xi$  sets the scale for the power spectrum, and the WMAP normalisation imposes  $\xi \sim (10^{15} \text{Gev})^2$  assuming  $g^2 \sim 10^{-2}$ . Any cosmic strings formed in this process have a tension  $G\mu \sim \xi M_p^{-2}$ , which is fortunately well below the current observable threshold.

*Axion monodromy.* An interesting proposal which has developed from the brane models we have been discussing relies on axion monodromy [54]. This requires a  $D5$ -brane to be present in a type IIB compactification, wrapping some two-cycle ( $\Sigma_2$ ) and carrying NS-NS flux on the worldvolume <sup>+</sup>. One can associate an axion to this flux through a term  $b = 2\pi \int_{\Sigma} B$ . Computation of the brane action in a particular compactification, results in a scalar (inflaton) potential that is linear in  $b$  (provided that it is larger than the size of the compact cycle), and therefore gives rise to a linear inflaton potential of the form of Eq.(2.5) (b) with  $p = 1$ . Such a potential is strongly disfavored from a field theory perspective, and therefore could be considered a signature of stringy physics. Since a relation between  $r$  and  $n_s$  exists for this model it is testable without knowing  $\mathcal{N}^*$ . For  $\mathcal{N}^* \sim 60$ , one finds  $r \sim 0.07, n_s \sim 0.975$  [54], within current WMAP bounds. Compactification of the model using  $D4$ -branes on a Nil-manifold results in a fractional power law potential of the form  $\phi^{2/3}$ , the predictions of which follow from Eq.(2.5) (a) with  $p = 2/3$ . Again, such models are disfavored in field theory\*. One other interesting feature is that, dependent on the details of the compactification, the potential may have a superimposed small oscillation from instanton effects, which would lead to an

¶ The KKLMMT model [72] corresponds to  $\beta = 0$ .

<sup>+</sup> An S-dual system involving NS5-branes can also be constructed, however the axion now arises from integration of the RR flux.

\* One caveat here is that linear (and fractional) models can be found in the SUGRA literature [58] and therefore the sub-leading corrections present in the axion monodromy framework will be important in breaking the degeneracy with such models.

oscillatory feature in the power spectrum [57] and more pronounced oscillatory features in the bi-spectrum [55, 56]. If, for example, the level of the effect was too small to affect the power-spectrum relations discussed above, but could be seen in the bi-spectrum, this combined evidence would be powerfully predictive.

*Tachyon models.* One of the simplest, and most popular, models is that of tachyon inflation, driven by the condensation of an open string mode on a non-BPS  $D$ -brane [74]. Early constructions were unable to satisfy observational bounds because the tachyon mass was too large, however once warped models were developed this constraint could be evaded [76, 75]. Although the action is non-linear, tachyon inflation does not generate large  $f_{nl}$  because inflation ends before (ultra)-relativistic effects become dominant. A step towards a concrete UV embedding of this theory was developed in [77, 78] where they considered a non-BPS  $D6$  in a geometry generated by  $D3$  flux. The scalar index was found to be  $0.94 < n_s < 0.97$  for a string coupling in the range  $0.1 < g_s < 0.34$ , which suggested that larger  $D3$ -flux would lead to better agreement with experiment.

*Non-local Inflation* Thus far the models we have considered have been in the context of low energy supergravity. In the case of  $D$ -brane and tachyon actions we have considered models which contain terms of higher order in  $X$ , but none of the models retains higher derivatives. Generally there will be an infinite tower of such higher derivatives which, at energies above the string scale, cannot be ignored. A radical proposal, referred to as non-local inflation, aims to study the effect of such a tower in a limited way. One example uses the action for the tachyon from a toy model of string theory, the P-adic string, where the world-sheet coordinates are restricted to the set of P-adic numbers<sup>‡</sup>. Other settings include the action for the tachyon derived from truncated cubic string field theory (CSFT) (see for example [80]), which can also only be considered as a toy model. If non-local effects are generic, studying these models *may* still tell us something interesting about possible stringy observables. A general non-local scalar field action takes the form  $\mathcal{L}_\phi = \phi G(\square)\phi - V(\phi)$ , where  $\square$  is the d'Alembertian operator, and  $G$  an arbitrary analytic function. Considering this Lagrangian, one discovers that inflation can proceed even if  $V$  is naively too steep for slow-roll to be supported, in a sense the additional derivatives act as friction terms. For the p-adic string,  $G(x) = -\gamma^4 \exp(-\alpha x)$ , and  $V(\phi) = \gamma^4 \phi^{p+1}/(1+p)$  where  $\alpha = \ln p/(2m_s^2)$  and  $\gamma^4 = (m_s^4/g_s^2)(p^2/(p-1))$ . The potential is naively too steep for large  $p$ , however the effects of the infinite number of derivatives leads to a consistent dual hilltop inflationary model [81, 82]. While it is too early to say that inflation is more natural once higher derivative effects are taken into account, it is an intriguing possibility. A final remark is that initial calculations suggest this model can give rise to large non-Gaussianity, with [81]

$$f_{nl}^{\text{eq}} = \frac{5(\mathcal{N} - 2)}{24 \ln p} \sqrt{pr} |n_s - 1|^{3/2} \frac{(p-1)}{(p+1)}, \quad r = \frac{(p+1)}{2p} |n_s - 1| e^{-\mathcal{N}|n_s-1|} \quad (3.15)$$

<sup>‡</sup> This assumption was argued to be relaxed in to consider any positive integer ( $p$ ) [79].

where  $r$  decreases for larger values of  $p$ ,  $r$  is unobservably small of order  $\mathcal{O}(10^{-3})$ ,  $f_{nl}$  clearly scales as  $\sqrt{pr}$ , and for large  $p, r$  becomes independent of  $p$ , allowing  $f_{NL}$  to become large. One issue is that determining the precise end of inflation (and hence precise observational parameters) requires knowledge of the dynamics of the fully non-linear regime, which is extremely difficult [83]. Interestingly the shape of the non-Gaussianity is different from DBI models, peaking on squeezed triangles, similar to the shape produced by multi-field models. This signature is interesting, but whether it can be distinguished from the multi-field models is unclear, and it may require knowledge of higher order statistics such as the trispectrum. The scalar index is red with current calculations giving

$$|n_s - 1| \sim \frac{4}{3} \left( \frac{m_s}{H} \right)^2 \quad (3.16)$$

and  $H > m_s$ . Such a condition is acceptable in this model because of the possible UV completion and, as discussed, is the source of the novel features present.

*Assisted inflation models.* A number of the models above may contain multiple fields when we move beyond the minimal scenarios. Typically a small number of fields are considered, both in order to keep the calculation tractable and because a larger number of fields implies more freedom and hence less opportunity for models to be probed by observations, or make robust predictions. In the limit where a very large number of fields are present however, interesting effects can occur. In certain models; many fields with potentials which are naively too steep to give rise to inflation can act in a collective, assisted manner enabling inflation to proceed [84, 85]. This also allows each individual field to travel sub-Planckian distances. Furthermore the collective behaviour can appear very similar to inflation sourced by one field moving over a much larger distance, and hence  $r$  can be large enough to be observable. From one point of view such scenarios appear natural, since the assisted behaviour relaxes the conditions each individual potential must satisfy, and they also have the potential to be testable and predictive, because the very large number of fields can enable a statistical approach to making predictions.

One such model of interest is N-flation [86], which considers a large number of axion fields, each paired with a modulus of the compactification, to act collectively to source inflation. With coupling neglected, each axion has a sinusoidal self-interaction potential of the form  $V_n(\phi_n) = \Lambda_n^4 (1 - \cos(2\pi\phi_n/f_n))$ , which appears like a quadratic potential when expanded around a minimum, with  $m_n = 2\pi\Lambda_n^2/f_n$ . Then, if the masses are identical, the theory is effectively that of a single field sourced by a quadratic potential of the form of Eq.(2.5) (b) with  $p = 2$  and hence  $n_s - 1 = -2/\mathcal{N}$ ,  $r = 8/\mathcal{N}$ . For  $\mathcal{N} \sim 60$ , and with  $f < m_{pl}$ , the number of axions required is typically thousands. The observational signature changes if the axion masses are not all identical, and a more realistic approach is to have masses distributed according to a Marcenko-Pastur probability distribution  $p(m^2) = \sqrt{(b - m^2)(m^2 - a)}/(2\pi m^2 \beta \sigma^2)$ , where  $a < m^2 < b$ ,  $a = \sigma^2(1 - \sqrt{\beta})^2$ ,  $b = \sigma^2(1 + \sqrt{\beta})^2$ ,  $\sigma^2 = \langle m^2 \rangle$  and  $\beta \sim 1/2$  is typical, and depends on the dimension of the Kähler and complex moduli spaces [87, 88]. Remarkably this

statistical approach is surprisingly testable. When comparing to observations we must also fix initial conditions for the various fields. One approach is to also do this randomly, and one finds average values for the spectral index are typically lower than with equal masses,  $n_s \approx 0.95$  for 50 e-folds, this being insensitive to the distribution from which the initial conditions are drawn.  $r$  is independent of the model parameters and given by the same expression as above and the non-Gaussianity negligible [89]. An intriguing recent development has been the observation that when the full axion potential is considered, a large local  $f_{\text{NL}}$  can be produced, with  $f_{\text{NL}}^{\text{loc}} \sim 10$  when all axions are taken to be identical, and  $f = m_{\text{pl}}$  [90], though in this case  $r$  is negligible and  $n_s$  slightly lower again, putting the model in tension with WMAP. This result has been calculated using the  $\delta N$  formalism, and may be altered in the light of numerical simulations [91], and should be tested in more realistic settings and mass distributions. We note that statistical approaches may well have a role to play when confronting other complicated string theory models with observation.

*M-theory models* A robust scenario in Heterotic M-theory reproduces the results of assisted power law inflation where the potential is exponential and  $a(t) \sim a_0 t^p$ . Inflation here occurs before moduli stabilisation and is driven by the non-perturbative dynamics of  $N$  five-branes along the orbifold direction [59]. Under a set of reasonable assumptions, the instanton generated scalar potential is always the steepest direction in field space, which would be unable to support single field inflation. The scalar index takes the expected form  $n_s = 1 - 2/p$  where  $p = N^3 + \dots$  which is used to fix the number of branes using the WMAP data. Whilst the  $R^4$  corrections are known, their implementation is difficult since they compete with higher order instanton effects - spoiling the simplicity of the model. However their inclusion could break the field theory degeneracy and point to a unique signature of M-theory. Moreover moduli stabilization and subsequent reheating in this model will no doubt further constrain the parameter space, and test the viability of such a scenario.

A related model arises with only a single five-brane wrapped on the orbifold [60], where the inflaton is identified with the real part of the five-brane modulus ( $x$ ). For  $x \ll 1$  the five-brane is localised near the visible sector, and inclusion of a FI-term in the *hidden* sector uplifts the stabilised vacuum to  $dS$ . Slow roll inflation (with no back-reaction) occurs in this regime with  $\mathcal{N} \sim \eta^{-1} \ln(x_i/x_f)$ , where  $x_i, x_f$  are the initial and final positions of the brane, and  $\eta$  is the slow roll parameter which can be expressed as a ratio of the fluxes arising from the superpotential. With  $\eta = 0.1, x_i = 10^4 x_f$  we find  $\mathcal{N} \sim 80$  and  $\mathcal{P}_s^2 \sim 10^{-10}$  which agrees with WMAP. Inflation ends once the five-brane dissolves into the visible sector via instanton transition, this in turn excites vector bundle moduli resulting in a shift of the cosmological constant. For larger values of  $x$  other moduli will be destabilised from their vacua, and may subsequently lead to a novel inflationary footprint.



### 3.3. Alternative models

Inflation is by far the most developed, and promising, theory for the origin of structure in the universe. But in the context of string cosmology, other scenarios exist which could be more natural. We briefly mention below some attempts to develop such alternatives.

*Ekpyrotic model.* The Ekpyrotic model is an alternative to inflation [92, 93, 94]. Instead of generating perturbations during an exponential expansion, successive  $k$  modes exit the horizon during a slow contraction. Predictions are made predominantly within an effective field theory, but the model can be embedded in Heterotic M-theory. In the original single field models the inflaton is associated with the distance between the two 5D ‘end of the world’ branes located at the orbifold fixed points  $\dagger\dagger$ , and has a steep negative potential, not directly derived from the theory. As the field evolves, the universe collapses and these branes approach one another. During the collapse the spectrum of  $\zeta$  perturbations is extremely blue and not phenomenologically viable [95], however an almost scale invariant spectrum can be produced in the Newtonian potential [105]. Standard hot big bang cosmology is recovered when these branes collide, and it is possible that scale invariant perturbations get imprinted on  $\zeta$ , but this requires going beyond the 4-dimensional description [96] and is a potential weakness of the model. An alternative suggestion is to consider a two-field model, the second field arising from the volume modulus of the internal dimensions [97, 98, 100]. If both fields have steep negative potentials parametrised by  $V_i = e^{-c_i(\phi)\phi}$ , then (in field space) the shape of the potential looks like a ridge. If the trajectory is finely tuned such that the inflaton rolls down this ridge, then the isocurvature perturbation produced during collapse is close to scale invariant. If the trajectory subsequently curves, either by the trajectory naturally falling off the ridge or by ‘bouncing’ off a boundary in field space, then this isocurvature perturbation can be converted into  $\zeta$ . The model predicts,  $n_s - 1 = \epsilon^{-1}(2 - \partial \ln \epsilon / \partial \mathcal{N})$  where the first term is blue-tilted and the second term red-tilted. Tensors are unobservable, which means that detection of almost scale invariant gravitational waves will rule out Ekpyrosis, and strongly favour the simplest inflationary models.

The conversion of isocurvature to curvature perturbations has a secondary effect, which is to produce a large value of  $f_{\text{NL}}$  in the squeezed shape typical of multi-field models. For the simplest case where the conversion occurs by naturally falling off the ridge,  $f_{\text{nl}} \sim -5c_1^2/12$ , where 1 labels the field which dominates at late times, and can be calculated using the  $\delta N$  formalism [99]. Clearly large non-Gaussianities can be generated if  $c_1 \gg 1$ , and moreover  $c_1 \gg 10$  is required for consistency of the spectral index with WMAP data, and therefore the level of non-Gaussianity is in severe tension with observation. In the case where the trajectories ‘bounce’, positive and negative values of  $f_{\text{NL}}$  are possible and the amplitude depends on how suddenly the bounce occurs [101, 103, 104]. Interestingly in search of a robust predictive signal, the authors have considered higher order statistics [102], and though no meaningful constraints currently

$\dagger\dagger$ The additional 6 dimensions being compactified on a small scale.

exist, future observations may probe the scenario in this way. A final comment on this scenario is that the initial conditions are extremely finely tuned. While mechanisms have been suggested to alleviate this tuning in a pre-ekpyrotic phase [106], it is hard to not to consider the evolution rather unnatural. On the other hand, because of the special initial conditions required to make the model work, in contrast to multi-field inflationary models, it is extremely testable and potentially predictive.

*String gas cosmology.* A novel program which both aims to understand the effect of the extended nature of strings on the early universe, and has attempted to replace inflation with an alternative mechanism of generating scale invariant perturbations, is that of string gas cosmology [107, 108]. This involves coupling the graviton and dilaton to a string gas (which may also include other degrees of freedom such as branes) and using T-duality to interchange winding and momentum modes. The model predicts a slightly red scalar index, but a blue tilt for gravitational waves which allows the theory to be ruled out. Interestingly the theory favours the Heterotic string due to existence of enhanced symmetries necessary for moduli stabilization.

*Pre-big bang.* Older models of the early (stringy) universe restricted themselves purely to the dilaton sector, at leading order in world-sheet and string loops (with  $V(\phi) = 0$ ). Application of generalised T-duality led to the existence of scale-factor duality which aimed to resolve the Big Bang singularity by replacing it by an epoch of high, but finite, curvature [109]. At early times, before this 'Big Bang', we find  $g_s \ll 1$  allowing us to probe the perturbative string vacuum without worrying about loop corrections. The coupling increases as we pass through this singularity until it becomes constant at late times. However this simple picture does not account for the observed perturbations, the dilaton perturbations leading to a strongly blue spectrum, instead one must consider a curvaton mechanism driven by an axionic field dual to  $B_{\mu\nu}$ . In turn this drives the production of both a graviton and dilaton background, where the dilaton mass can be  $m \geq 10^{-23} \text{eV}$ , which is detectable (in principle). The curvaton potential is assumed to be quadratic, in which case the predictions are the same as canonical  $m^2\phi^2$  inflation and satisfy the WMAP data. Interestingly the type I string is favoured over the Heterotic string in such models due to the difference in primordial magnetic seed production.

### 3.4. Reheating

Reheating is a significantly less developed topic in comparison to inflationary model building, but hugely important. Indeed in many instances there are only vague ideas as to the existence/location of the standard model sector. As we have already discussed this lack of post-inflationary knowledge means  $\mathcal{N}$  is not fully determined, and hence observational predictions ambiguous. Reheating is also interesting in its own right, and although the energy scale involved is significantly smaller than that associated with inflation, one may still hope that there is sensitivity to the extended nature of the superstring.

A landmark paper [110] considered the case of (warped) closed and open string reheating. The closed string sector was purely in the supergravity limit, and despite leading to interesting results, they argued that it was hard to distinguish between string theory and Kaluza-Klein physics unless one could examine the  $H/M$  expansion order by order. In the open string case, the strings were argued to redshift like matter. Both approaches further suggested the formation of long string networks during inflation, which could be detectable in the CMB.

More recent papers discuss the case of Kähler inflation in two different classes of closed string model [111], depending on whether the inflaton is the size of a blow-up mode of the Calabi-Yau (BI model), or whether it is the size of the  $K3$  fibre in Fibre inflation (FI model). Both cases involve the leading order  $\alpha'$  corrections, although  $g_s$  corrections are argued to be decoupled from the theory. The results indicate that there will always be hidden sectors present, that the BI model requires a much higher level of fine-tuning than FI since the hidden sector must wrap the same four-cycle as the inflaton in the former scenario - however despite the higher degree of tuning, the FI scenario leads to a small reheat temperature which is incompatible with TeV scale SUSY. As such, it should be disfavored. The final conclusion was that the hidden and visible sectors are not directly coupled, and the degrees of freedom in the visible sector cannot be more strongly coupled than its hidden sector counterparts. They further identified two generic problems with such a reheating mechanism; i) Inflationary energy will be transferred to the hidden sector. This is not a problem if the hidden sector degrees of freedom are relativistic, but does require a curvaton mechanism to generate the perturbations in the visible sector. ii) There may be overproduction of hidden sector dark matter, which would spoil BBN.

Reheating of the axion monodromy scenario has also been explored, at least in the IIA framework where the  $D4$ -brane unwinds [112]. The  $D4$ -brane passes through a  $D6$ -brane, however the open string modes present during the collision act as a braking force. This interaction was described by a toy field theory model, and suggested that no energy was transferred during these collisions. All the reheating energy is dumped into the final collision event, resulting in a high reheating temperature. However backreactive effects were not considered, and may spoil the simple field theory picture.

### 3.5. Cosmic Strings

A striking prediction of several string scenarios is the formation of cosmic super-strings during or even *after* inflation [114]. Such objects confront observation in a number of ways. First they contribute to primordial density perturbations, and CMB analysis indicates that they cannot account for more than 11% of the power [122], limiting their tension to  $G\mu < 2.1 \times 10^{-7}$  [123]. Moreover, the vector-mode perturbations they source will lead to CMB polarisation potentially observable by the Planck satellite or future missions [124]. They can strongly gravitationally lens background objects in a distinctive way, but as yet no candidate lensing event has been seen, and through vector

perturbations they will rotate images observed in weak lensing surveys [125]. The most promising way in which they can be detected, however, is through a gravitational wave signal produced by cusps on the strings, potentially detectable for tensions as low as  $G\mu \sim 10^{-10}$ , though whether this signal will be observable by LIGO, LISA or BBO is model dependent [115]. Current limits come from pulsar timing bounds, which lead to  $G\mu < 1.5 \times 10^{-8} c^{-3/2}$ , where  $c$  is the number of cusps per string loop.

The discovery of evidence for cosmic super-strings would be extremely powerful evidence for string theory. This requires, however, that they be distinguished from standard cosmic strings, and this is extremely challenging (see for example [119]). One important difference is that intersecting field theory strings recombine with probability  $P$ , passing through one another with probability  $1 - P$ . Numerical simulation, along with theoretical calculation, suggests that  $P \sim 1$  to a remarkably high degree for strings of the same type. For F-F strings it turns out that  $P \sim g_s^2$ , suggesting they will pass through one another rather than reconnecting. If  $P$  and  $\mu$  can be determined through gravitational wave observation, these possibilities can be distinguished. For F-D or D-D interactions, the value of  $P$  is less constrained, valued in the range  $P \in 10^{-3} \dots 1$ . Therefore the perturbative F-F interaction could provide the best direct evidence for string theory [114, 116]. Cosmic super-strings of different type may also combine to form  $(p, q)$ -strings or even networks [117, 118]. Such objects have a distinctive tension spectrum which is remarkably difficult to recreate using perturbative field theory.

In Heterotic M-theory one can consider three different types of cosmic strings [120], a membrane wrapped on the  $x^{11}$  direction, a five-brane wrapped on a four-cycle ( $\Sigma_4$ ) of the internal space or a five-brane wrapped on the product space  $\Sigma_3 \times x^{11}$  - where  $\Sigma_3$  is a three-cycle. Such strings are formed after the inflationary phases discussed in [59, 60]. The membrane tension is too large and is strongly dis-favoured. The only stable five-brane string is the one wrapped on  $\Sigma_4$  because one must turn on a gauge field to cancel the anomaly term, which must live on the boundary and only the brane wrapped on  $\Sigma_4$  can be stabilised. Such a string can be superconducting and generates seed magnetic fields that are coherent on *all* cosmological scales. Those fields at large scales cannot be amplified by a dynamo mechanism and therefore will have a weak, coherent amplitude, which may be detectable in future experiments.

#### 4. Discussion

In this paper we have provided a critical, non-exhaustive, review of the current observational status of string theory using cosmological data. We have emphasised that inflationary model building has been a success, in the sense that string theory can accommodate inflation, and that the footprints of many different models are testable and conform with WMAP data. Planck will offer a considerably more stringent test, and will undoubtedly rule out many models. Most models, however, exist as field theories and do not make direct predictions for how stringy corrections lead to shifts in observables. Never-the-less we have reviewed a number of signals, such as non-Gaussianity, and

relations between parameters, which might in combination with other considerations be evidence for stringy models. There are even some indications that inflation might be more natural in certain stringy settings. Alternatives to inflation are less well developed, but may also offer predictions which allow them to be probed observationally.

Reheating in inflationary models which includes stringy corrections will be different to those in field theory, and have been examined in several specific instances with important phenomenological consequences. However much more work needs to be done, particularly in M-theory, if these models are to be falsifiable. The cleanest signal for string theory still remains the detection of a cosmic superstring through the colliding F-F channel, although more work needs to be done on understanding how such strings scale during the reheating phase, and whether the dynamics of network formation is likely to be important.

We hope our review has highlighted the need for future work in a number of key areas. First, we note that parts of a model are often studied and compared with observation in isolation. For example inflationary predictions, and subsequent evolution including the reheating scale are generally treated separately, while in reality the later impacts on the former through  $\mathcal{N}^*$ . Likewise the production of cosmic strings is often treated separately and compared with observation independently of inflationary constraints, while the presence of both will alter the constraints considerably. This points to the need for a more holistic treatment, and for work on complicated questions. Another key issue is correctly accounting for the presence of more than one inflationary field, typical of complex models. Perhaps most importantly is the issue of the inclusion of higher order corrections, leading to robust cosmological tests.

Aside from these issues there exist other ways in which string theory could be cosmologically tested. One proposal is to embed inflation in non-geometric flux compactifications. Since non-geometric fluxes arise from multiple T-dualities, they are inherently stringy, therefore observables should directly probe the underlying theory. Work along these lines [126] determined a no-go theorem for massive IIA with metric flux. Only the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  case evaded this stringent theorem, but did not lead to slow-roll inflation. Future models are likely to evade this theorem, and it will be interesting to determine their footprints.

Finally, we note that there are many other areas of string cosmology we have not been able to discuss, such as corrections to the power spectra due to new physics at high energy scales [121], or effects from large extra dimensions. It is likely that work on several fronts will be necessary to determine which footprints are inherently stringy. Although thus far there are no truly convincing models or models with signals uniquely predictive of string theory, there remains much work to be done and the future for this field remains bright.

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